

On the Possibility of Experimental Verification of Some Localization Theory Predictions

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Abstract

The spatial non–locality (dispersion) of the transport equations results in a nonlinear dependence of the voltage drop U on the distance between the points of measuring. Therefore the results of the usual two–probe measurements of the conductivity depend essentially on the relation between the sample linear size L and the spatial dispersion scale R of the generalized diffusion coefficient $D(q, \omega)$. This makes it possible to get information on the character of the spatial non–locality of $D(q, \omega)$ in the Anderson localization regime and, in particular, on the value of the correlation multifractal dimension D_2 of the electron wave functions near the mobility edge.

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1. In the last few years considerable attention has been focussed on the problem of spatial dispersion (non–locality) of the kinetic coefficients of disordered systems in the Anderson localization regime [1–6]. The cause of such interest was the realization of the fact that the character of the $q\omega$ –dependence of the generalized diffusion coefficient $D(q, \omega)$ near the mobility edge is intimately connected to critical behavior of the electron wave functions and, in the end, is determined by the scenario of the metal–insulator transition [1, 3]. Indeed, the Berezinskij – Gorkov localization criterion [7] requires that in the localized phase $D(q, 0)$ vanish simultaneously for all values of the wave number q , and, according to the one–parameter scaling hypothesis, the relation

$$D(t; q, \omega) = b^{2-d} D(b^{1/\nu} t; bq, b^d \omega), \quad 2 < d < 4 \quad (1)$$

must be satisfied [8]. Here $t = (\mathcal{E} - \mathcal{E}_c)/\mathcal{E}_c$ is the distance to the mobility edge \mathcal{E}_c , b is a scaling factor, d is the space dimension, ν is the correlation length critical index.

There are two qualitatively different versions of the critical behavior of $D(q, \omega)$ that obey these general requirements.

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- According to Chalker's hypothesis [1], the multifractal structure of the electron wave functions near the Anderson transition ($t \rightarrow 0$ and/or $\omega \rightarrow 0$) leads to an anomalously *strong* spatial dispersion of the generalized diffusion coefficient, whose scale is $R = \min(\xi, L_\omega) \rightarrow \infty$, where $\xi \propto |t|^{-\nu}$ is the correlation length, $L_\omega \propto \sqrt{D(\omega)/\omega} \propto \omega^{-1/d}$ is the electron diffusion length in a time of $\sim 1/\omega$ [1, 2]. Depending on the relation between ξ , L_ω and q Chalker [1] distinguishes four main types of the asymptotic behavior:

$$D(q, \omega) = D_0 \left(\frac{l}{R} \right)^{d-2} \begin{cases} 1, & qR \ll 1, \\ (qR)^{d-2-\eta}, & qR \gg 1, \end{cases} \quad R = \min(\xi, L_\omega). \quad (2)$$

Here D_0 is the Drudian diffusion coefficient, l is the mean free path, η is a critical index related to the correlation multifractal dimension of the wave functions D_2 ($\eta = d - D_2$) [2].

- On the other hand, Suslov's symmetry approach to the localization theory [4] predicts *suppression* of the spatial dispersion of the diffusion coefficient in the vicinity of the Anderson transition down to atomic scales $\sim \lambda_F$. More recently, this conclusion was confirmed within the framework of a generalized formulation [5, 6] of the self-consistent Vollhardt — Wölfle theory [11]. According to [5, 6], in the Anderson localization regime

$$D(q, \omega) = \frac{D(t, \omega)}{1 + (qR)^2}, \quad qR \ll 1, \quad (3)$$

where the non-locality scale $R \propto \sqrt{D_0 \tau} |D(t, \omega)/D_0|$ and decreases as $D(t, \omega) \propto [\min(\xi, L_\omega)]^{2-d}$ until saturation is reached at $R \sim \lambda_F$.

In the survey by Suslov [3] it is pointed out that the absence of anomalously strong spatial dispersion of the generalized diffusion coefficient near the mobility edge does not contradict the concept of multifractality of the electron wave functions, it only indicate that the equality $\eta = d - 2$ (or $D_2 = 2$) should be satisfied. The well-known Wegner's result $\eta = 2\epsilon$ ($\epsilon = d - 2 \ll 1$) [9] is directly connected with the critical behavior of the inverse participation ratio. At the same time, the relation between this quantity and $D(q, \omega)$ used in [1, 2, 10] cannot be considered correct for several reasons.¹ Therefore, in our opinion, the above contradiction is only apparent. The same may be said about the results of numerical modelling $\eta = 1.2 \pm 0.15$, $\eta = 1.3 \pm 0.2$, $\eta = 1.5 \pm 0.3$ [2] and $\eta = 1.3 \pm 0.2$ [10] obtained by different methods for $d = 3$.

This dilemma, touching upon the fundamental notions of the Anderson localization, calls for both theoretical and experimental solution. In

¹The detailed discussion of the basic arguments pro and con hypothesis $\eta = d - 2$ may be found in [3].

this letter we derive a material equation relating the current density in the spatially nonuniform case to the experimentally measured difference of the electrochemical potentials, and propose a measuring scheme that allows one to obtain information on the degree of non-locality of the diffusion coefficient of charge carriers.

2. The voltage drop U measured in the spatially nonuniform case is equal to the difference in electrochemical potential $\Delta U = \Delta\zeta/e = \Delta\varphi + \Delta\mu/e$ between the corresponding points of a conductor. Therefore in calculating the current density it is necessary to take into account the response of the system to both the mechanical perturbation (electrical potential φ) and the thermal one (chemical potential μ) caused by the nonuniform electron distribution induced in the conductor.²

Applying the general equations of the linear response theory [12, 13] to the problem under consideration, we obtain a material equation relating the current density to the electrochemical potential gradient. for $q \ll k_F$ its Fourier representation has the form

$$j(q, \omega) = -iqen_FD(q, \omega)\zeta(q, \omega) = -iq\sigma(q, \omega)U(q, \omega)/e, \quad (4)$$

where n_F is the density of states at the Fermi level, $\sigma(q, \omega)$ is the measured electrical conductivity connected with $D(q, \omega)$ by the Einstein relation. It should be emphasized that $\sigma(q, \omega)$ is not Kubo's kinetic coefficient

$$\tilde{\sigma}(q, \omega) = e^2 n_F \frac{D(q, \omega)}{1 + i(q^2/\omega)D(q, \omega)}, \quad (5)$$

which, unlike $\sigma(q, \omega)$ (see Eq. (4)), relates the total current density to the electrical field acting in the system. It is in the uniform case ($q = 0$) only that $\sigma(0, \omega) = \tilde{\sigma}(0, \omega)$, and Eq. (5) coincides with the Einstein relation.

Eqs. (4) and (5) describe the non-local linear response of a spatially unbounded and homogeneous system. In the general case, the non-locality of material equations is of a more complicated character. In the typical experimental situation, the sample has the shape of a plane-parallel slab of thickness L . In this case the diffusion propagator of charge carriers $\tilde{G}(x, x'; \omega)$ is a solution of the equation

$$-i\omega\tilde{G}(x, x'; \omega) - \frac{\partial}{\partial x} \int_{-L/2}^{L/2} \tilde{D}(x, y; \omega) \frac{\partial}{\partial y} \tilde{G}(y, x'; \omega) dy = \delta(x - x') \quad (6)$$

²The nonuniform spatial electron distribution results not only in perturbation of the electrical field in the conductor, which is involved in φ , but also in the appearance of the diffusion term in the measured total current density. Just the latter term is the response of the system to the thermal perturbation.

with the open boundary conditions

$$\widetilde{G}(x, x'; \omega)|_{x, x' = \pm L/2} = 0. \quad (7)$$

The integral kernel of this equation (non-local diffusion coefficient) enters into the material equation that relates the electrical current to the electrochemical potential gradient (voltage drop)

$$I(x) = -e^2 n_F S \int_{-L/2}^{L/2} \widetilde{D}(x, x'; \omega) \frac{\partial}{\partial x'} U(x') dx', \quad (8)$$

where $S \propto L^{d-1}$ is the area of the sample cross section. The solution of the boundary problem (6), (7) in the absence of spatial dispersion ($\widetilde{D}(x, x'; \omega) = D(\omega)\delta(x - x')$) is given by well-known method of images [14]

$$\begin{aligned} \widetilde{G}(x, x'; \omega) &= \sum_{n=-\infty}^{+\infty} [G(x - x' + 2nL; \omega) - G(x + x' + (2n + 1)L; \omega)], \\ G(x; \omega) &= \frac{1}{L} \sum_q \frac{\exp(iqx)}{-i\omega + q^2 D(\omega)}. \end{aligned} \quad (9)$$

Here $G(x, \omega)$ is the diffusion propagator of charge carriers in a spatially unbounded homogeneous system. We assume that the solution (9) holds true in the presence of spatial dispersion if its scale is small in comparison with the sample size ($R \ll L$). In this case the integral kernel of the material equation (8) is expressed, by analogy with (9), in terms of the generalized diffusion coefficient $D(q, \omega)$ of the spatially unbounded and homogeneous system

$$\widetilde{D}(x, x'; \omega) = \frac{2}{L} \sum_{n=0}^{+\infty} [D(\bar{q}_n, \omega) \sin \bar{q}_n x \sin \bar{q}_n x' + D(q_n, \omega) \cos q_n x \cos q_n x'], \quad (10)$$

where $\bar{q}_n = 2\pi n/L$ and $q_n = \pi(2n + 1)/L$ are discrete values of the wave numbers.

Taking into account that the voltage drop $U(x)$ is an odd function of x , and the current strength $I(x) = I$ is constant along the studied sample, we will represent them through the corresponding Fourier series. Then, upon substitution of (10) in the material equation (8), it is not difficult to find the expression for the voltage drop Fourier coefficient U_n . Finally the x -dependence of the voltage drop can be presented as the following Fourier series:

$$U(x) = \frac{4I}{LSe^2 n_F} \sum_{n=0}^{+\infty} \frac{(-1)^{n+1}}{q_n^2 D(q_n, \omega)} \sin q_n x, \quad |x| < L/2. \quad (11)$$

3. When the diffusion coefficient is independent of the wave number q_n the series (11) gives, for a finite-size conductor, the usual conductance definition $g(L, \omega) = L^{d-2} e^2 n_F D(0, \omega) = L^{d-2} \sigma(\omega)$ and describes the linear x -dependence of the voltage drop $U(x)$ within the sample ($|x| < L/2$). The spatial non-locality of the generalized diffusion coefficient $D(q_n, \omega)$ changes the conductance definition as:

$$\frac{1}{g(L, \omega)} = \frac{8}{L S e^2 n_F} \sum_{n=0}^{+\infty} \frac{1}{q_n^2 D(q_n, \omega)} \quad (12)$$

and results in a nonlinear x -dependence of the voltage drop (11). Thus, information on the spatial dispersion of the generalized diffusion coefficient of charge carriers $D(q, \omega)$ can be obtained by measuring the nonlinear part of $U(x)$ (11):

$$\Delta U(x) = U(x) - \frac{2x}{L} U(L/2). \quad (13)$$

Consider a sample in the form of a plane-parallel slab of thickness $L \gg R$ (R is the spatial non-locality scale) with ideal ohmic contacts on the opposite surfaces and with two potential measuring probes which symmetrically located at a distance x from the mid-plane (see Fig. 1(a)). It is best to situate the measuring probes near the x_{\max} points where $\Delta U(x)$ reaches its maximum value ΔU_{\max} . Another pair of potential probes, the role of which is played here by the ideal ohmic contacts, are necessary for measuring the non-linear part of the voltage drop (13).

Substituting (3) into (11), (13) yields $\Delta U(x) = 0$. Strictly speaking, this equality is fulfilled in the case of ideal compensation of the linear part of the voltage drop. In any case, however, the measured signal is small in parameter $R/L \ll 1$. The nonzero contribution to $\Delta U(x)$ comes only from the nontrivial q -dependence of the generalized diffusion coefficient from (2). Fig. 1(b) shows typical dependences of $\Delta U(x)$ calculated using linear interpolation between Chalker's asymptotic formulas (2) for the inverse diffusion coefficient $1/D(q, \omega)$ at $qR \ll 1$ and $qR \gg 1$. In this case x_{\max} does not depend on the scale of spatial non-locality of $D(q, \omega)$ and assumes values on the interval $x_{\max} \approx (0.70 \div 0.76)L/2$ for $\eta = 1.1 \div 1.5$. The non-linear part ΔU_{\max} calculated at these points has the following asymptotic behaviors

$$\Delta U_{\max} \propto \left(\frac{R}{L}\right)^\eta \propto \begin{cases} \omega^{-\eta/d}, & L_\omega \ll \xi, \\ \xi^\eta \propto |t|^{-\nu\eta}, & L_\omega \gg \xi, \end{cases} \quad L_\omega, \xi \ll L. \quad (14)$$

Two ways for measuring the signal (14) may be suggested. The first is to analyze the frequency dependence of $\Delta U_{\max}(\omega)$ in a sample with a fixed

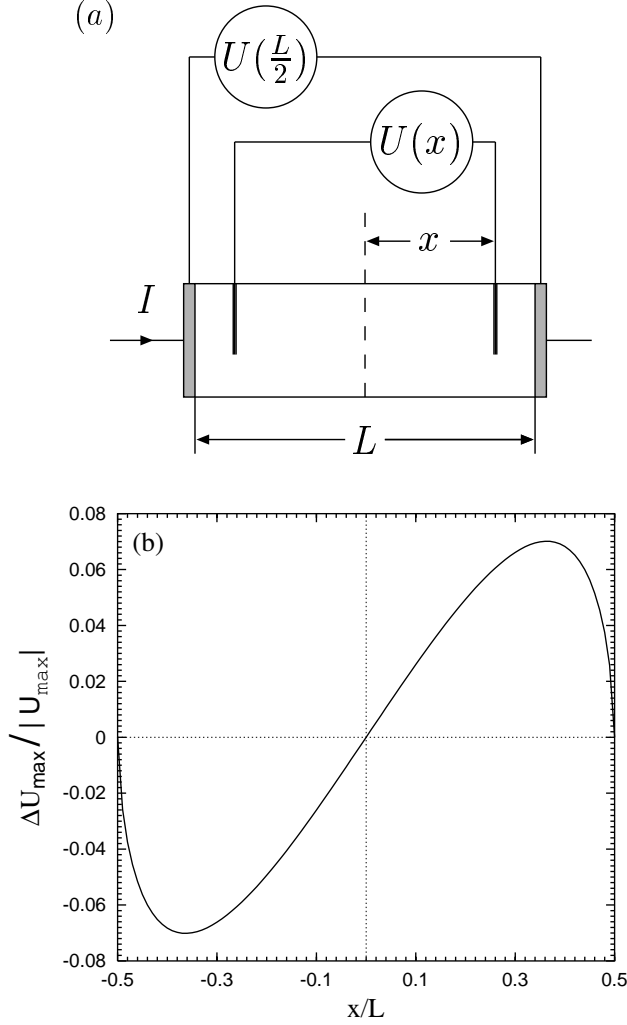


Figure 1: (a) The scheme of the measurement of the nonlinear part of the voltage drop (here is depicted a longitudinal section of investigated sample); (b) Typical dependence of the nonlinear part of the voltage drop on the potential probe position x . Curve is calculated using Chalker's asymptotic formulas (2) for $\eta = 1.3$ and for $R/L = 0.1$.

disorder level in a small enough vicinity of the mobility edge on the metallic side of transition ($|t| \ll 1$). According to (14) and the predictions of Ref. [1], as the frequency decreases, an increase in $\Delta U_{\max} \propto \omega^{-\eta/d}$ ($\omega \gg \omega_c$) should be observed until saturation is reached at $\Delta U_{\max} \propto (\xi/L)^\eta$ ($\omega \ll \omega_c$). The typical frequency ω_c is determined by the relation $L\omega_c = \xi$ or $\hbar\omega_c \approx \lambda_F^{-d} n_F^{-1} |t|^{\nu d}$, where λ_F is de Broglie's wave length at the Fermi level.

The second way is to measure the dependence of ΔU_{\max} (14) on a dimensionless distance t to the mobility edge \mathcal{E}_c at a fixed frequency ω . The stress-tuning technique [15] seems to be the simplest method of changing t in the vicinity of \mathcal{E}_c . A very suitable material for such measurements is Si:P, in which this technique makes it possible to attain $|t| \approx 10^{-3}$ [15]. According to (14) and the predictions of Ref. [1], as t decreases, an increase in

$\Delta U_{\max} \propto t^{-\nu\eta}$ ($t \gg t_\omega$) should be observed until saturation is reached at $\Delta U_{\max} \propto (L_\omega/L)^\eta$ ($t \ll t_\omega$). Here the typical value t_ω is determined by the relation $L_\omega = \xi$ or $\lambda_F^d n_F \hbar \omega \approx |t_\omega|^{\nu d}$.

Such dependences obtained by Eqs. (11), (13) using the interpolation $R^{-1} = \xi^{-1} + L_\omega^{-1}$ for the non-locality scale, are plotted in Fig. 2(a,b). Since in a sufficiently small vicinity of the mobility edge the non-locality scale of $D(q, \omega)$ (2) takes on anomalously large values $R = \min(L_\omega, \xi)$, the value of ΔU_{\max} (14) is quite accessible for measurement. The estimates show that for Si:P samples with the typical phosphorus concentrations $n_P \approx 10^{18} \text{cm}^{-3}$ and for really attainable values $|t| \approx 10^{-2} \div 10^{-3}$, similar dependences of ΔU_{\max} (see Eq. (14) and Fig. 2) should be observed in the frequency region accessible for probe measurements. For example, at $t \approx 10^{-3}$ the typical frequency value is $\omega_c \approx 10^3 \div 10^4 \text{s}^{-1}$ (see Fig. 2(a)), and the corresponding correlation lengths are of order $L_{\omega_c} \sim \xi \sim 10^{-3} \div 10^{-2} \text{cm}$.

So, we believe that the existence [1,2] or suppression [4–6] of the anomalous spatial dispersion of the generalized diffusion coefficient near the Anderson transition can be verified experimentally.

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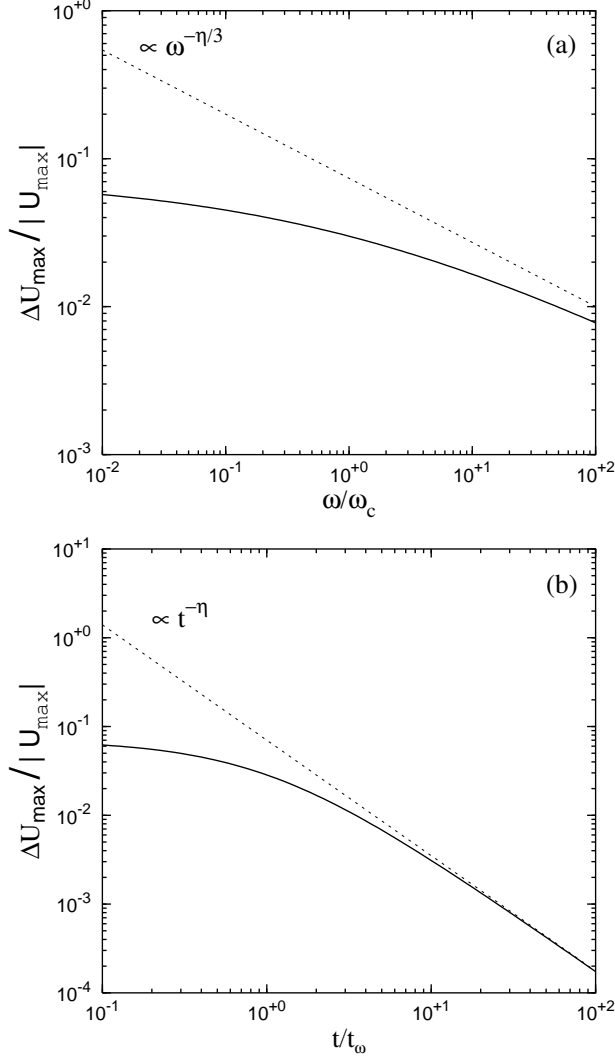


Figure 2: Typical dependences of ΔU_{\max} on the frequency, ω , (a) and dimensionless distance from the mobility edge, t , (b) calculated by Eqs. (11), (13) using Chalker's expressions (2) for $\eta = 1.3$. Dashed straight lines depict the asymptotic behavior predicted by Eq. (14).

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